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SUMMARY

This final report describes research on stochastic and adaptive systems by faculty and students of the Decision and Control Sciences Group of the M.I.T. Laboratory for Information and Decision Systems (formerly Electronic Systems Laboratory) with support provided by the United States Air Force Office of Scientific Research under Grant AFOSR 77-3281B. The Grant Monitor was Charles L. Nefzger, Major, USAF. The time period covered by this report is February 1, 1979 to January 31, 1980.

Substantial progress is reported in the areas of nonlinear filtering, stochastic control, adaptive control and stochastic adaptive control.

Introduction

The research we have conducted over the past several years and in particular during the period February 1, 1979 to January 31, 1980 has been concerned with fundamental aspects of controlling linear and non-linear stochastic systems in the presence of measurement and parameter uncertainties. In case the uncertainties reside in the state description of the physical system and measurements then we refer to the control problem as a stochastic control problem. If in addition there are parameter uncertainties then the problem is referred to as an adaptive control problem. This is because in addition to state estimation some form of parameter identification scheme will be needed and almost always the control and estimation-identification functions will interact in a non-trivial way. The simplest such example would be to try to control a plant which can be described by an integrator with an unknown gain and it is necessary to control this plant in the presence of additive white gaussian noise uncertainties in the state and measurement process. The performance of the controller is judged by setting up a quadratic performance criterion.

A subproblem of the stochastic and adaptive control problem is the state estimation problem. Suppose for a moment we make the assumption that there are no parameter uncertainties present in the dynamical description of the state of the system. Then all the probabilistic information that one can extract about the "state" of the system on the basis of noisy measurements of the state is contained in the conditional probability density of the state given the observations. Indeed this is the probabilistic state of the joint physical-measurement system. The recursive computation of the probabilistic state is the state estimation problem. If this could be done, then one could look for the best controller as a function of this probabilistic state, best being judged in terms of a suitable performance criterion.

During the current grant period we have made important progress on many aspects of the overall problem we have discussed above.

This report is divided into three main sections:

(i) Linear and Non-linear State Estimation

(ii) Stochastic Control

(iii) Adaptive Control

(iv) Stochastic Adaptive Control ,

corresponding to the subdivision of the control of uncertain systems we have made above.

This work was carried out under the joint direction of Professors M. Athans and S.K. Mitter. They were assisted by Dr. L. Valavani, Professor John Baras (visiting from University of Maryland and partially supported by the grant), Mr. D. Ocone (research assistant), Mr. T. Pappas (research assistant) and Mr. L. Vallot (research assistant).

2. Linear and Non-linear State Estimation

A general non-linear state estimation problem can be described as follows: Let the state x_t of the stochastic system have a dynamical description

(1) $x_t = x_0 + \int_0^t f(x_s)ds + \int_0^t g(x_s)dw_s$, where f and g are sufficiently smooth functions and dw_s is f white gaussian noise, and let the measurement equation be

(2) $y_t = \int_0^t h(x_s)ds + \eta_t$

where η_t is the integral of white Gaussian noise and h is also a smooth function. Both the processes x and y can be vector processes in which case f is a vector-valued function and g is matrix-valued.

The problem of non-linear state estimation is to recursively (in real-time) compute the conditional density $p(x_t|y_s\ ,\ 0 \leq s \leq t)$. The most celebrated special case of this problem is when the functions f, g, h are linear in x and this problem was completely solved by Kalman. In this case the conditional density is Gaussian and hence can be parametrized by its mean and covariance and differential equations for the evolution of the mean and covariance functions in time can be obtained. Furthermore under appropriate hypotheses of controllability and observability the resulting state estimator can be shown to be asymptotically stable.

An approach towards obtaining the solution to this problem is the "innovations approach", first proposed by Bode-Shannon in the early fifties and later developed by Kailath and his students. In this approach, one forms the so-called innovations process

(3) $\nu_t = y_t - \int_0^t \hat{h}(x_s)\ ds$, where ^ denotes conditional expectation, which

can be shown to be integral of white noise, and the estimator can then be computed based on the innovations process. A conjecture due to Kailath and Frost which has been open since 1967 related to the innovations process has recently been solved by Professor Mitter in collaboration with Dr. D. Allinger of the Mathematics Department at M.I.T. [1]. The conjecture was to the effect that the innovations process contained the same information as the observations process, or in more technical terms whether the σ-field generated by the observations equalled the σ-field generated by the innovations.

For non-linear problems, in general the innovations process cannot be effectively computed. It turns out that the correct object one should try to recursively compute is not the conditional density $p(x_t|y_s,\ 0\leq s\leq t)$, but an unnormalized form of it. If we denote this by $q(t,z,y_0^t)$ then it can be shown that q satisfies a bi-linear stochastic partial differential equation which depends on the observations $\{y_s,\ 0\leq s\leq t\}$ *and not on the innovations.* Furthermore we have been able to find classes of examples of non-linear estimation for which we can explicitly solve this equation. The systematic study of this equation, its robust form and its analogy to problems of quantum physics has been carried out by Professor Mitter jointly with Mr. Daniel Ocone (Ph.D. student in Mathematics Department and supported by this grant) and Professor John Baras of the University of Maryland who visited M.I.T. during the fall term of 1979 and was partially supported by this grant. Details of this work is reported in [2] and several other publications are in preparation.

In other work related to non-linear estimation, Professor Mitter in conjunction with Mr. Daniel Ocone has obtained multiple integral expansions for representations of conditional statistics of x_t, given the observations $\{y_s|0\leq s\leq t\}$. This work was reported in [3] and further details can be found in the forthcoming dissertation of Daniel Ocone.

Finally, a new approach to obtain estimates better than the linear minimum variance estimate for linear stochastic estimation problems with multiplicative noise is being investigated by Professor Mitter and Mr. Lawrence Vallot (graduate student, partially supported by this grant). This approach consists of generating new observables based on the observation y_t and the linear minimum variance estimate and improving the estimate based on these new observables. This work will be reported in the S.M. Thesis of Mr. L. Vallot.

The work done during the current grant period promises to have important practical consequences. For the first time, a systematic approach to obtaining sub-optimal estimators using perturbation theory appears to be feasible. We believe, this work would also lead to adequate analysis of the convergence properties of the extended Kalman filter and other successive linearization techniques.

Most of the major advances in non-linear estimation over the past two to three years have been done by the M.I.T. group or people who have visited the M.I.T. group (such as J. Baras, V. Benes, M.H.A. Davis, E. Wong, E. Pardoux).

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3. Stochastic Control

Professor Mitter assisted by Mr. Thrasyvoulos Pappas (research assistant) has continued his work on a geometrical theory of stochastic control. This work is a generalization of the work of W.M. Wonham on a generalized theory of deterministic linear multivariable control systems [1]. An important role is played in this theory by the concepts of (A,B) invariant and controllability subspaces.

The basic model is that of a linear multivariable system in state-space form which is perturbed by additive white Gaussian noise. In addition the measurements are also corrupted by additive white Gaussian noise and it is desired to regulate certain other output variables by means of linear constant feedback of the estimated state of the system. Regulation here is understood to mean that the variance of the variables to be regulated remains bounded. Preliminary work on this problem was done by Snyders and Wonham [2]. They obtain certain sufficient conditions for regulation to be possible. These sufficient conditions were obtained by relating this problem to the restricted regulator problem.

Recent work by L. Shumaker and J.C. Willems (as yet unpublished) suggests that it would be possible to obtain necessary and sufficient conditions for regulation based on the concept of (C,A) invariant subspaces. Furthermore, based on our work on state estimation it appears that we can introduce a concept of stochastic observability which will have an important role to play here.

Our work on state estimation reported in the previous section also has implications in stochastic control. It appears that for stochastic control in the presence of measurement uncertainties the control function can always be chosen as a feedback function of the (uncontrolled) unnormalized conditional densities.

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4. Adaptive Control

During the past year the adaptive control research accelerated, due primarily to the addition of Dr. L. Valavani in the research team. Our long range objective is to develop a methodology of design for adaptive control systems, by attempting to unify promising concepts based upon hyperstability theory and stochastic optimal control, respectively, with some common sense control engineering techniques. The reason for our relative optimism is due to the fact that during the past two years a better theoretical understanding of "model reference" adaptive control techniques has occured together with a unification of certain, hitherto distinct, adaptive control methods. In addition, a more fundamental understanding of robustness properties of multivariable control systems has taken place through the use of singular value diagrams. We feel that the time may be "ripe" for significant advances in the theory of adaptive control.

In the remainder of this section we summarize our progress to date in the following four areas:

(a) Stable Adaptive Schemes

(b) Convergence Properties of the Adaptive Process

(c) Stochastic Adaptive Control

(d) Adaptive Dual Control Studies .

Stable Adaptive Schemes

In recent years there has been considerable activity in the design of adaptive controllers, with special emphasis on their stability properties. Many different adaptive schemes were suggested and the question of asymptotic stability became of critical importance, particularly since in most of these schemes auxiliary feedback adaptive signals--which could conceivably become unbounded--were used in both plant and reference model. More recently, global asymptotic stability proofs have become available for some adaptive controllers--primarily those dealing with deterministic systems.

A detailed assessment of the various schemes proposed so far--both discrete and continuous--is given in a paper under preparation by Dr. Valavani [1]. It is shown that the stability proof of the various schemes derived by different methods can be given in a unified manner, using a generic model of the error equations. The paper also deals with the generalization of this model to a more abstract setting, thus providing the necessary insights towards new adaptive control schemes with fewer restrictive assumptions. Further, the particular adaptive schemes described in the literature are derived by applying special modifications to this general model. Work on the implications of such a unification to <u>optimal</u> adaptive schemes--as compared to the model reference schemes considered so far--is currently in progress and will be reported at a later time.

<u>Convergence Properties of the Adaptive Process</u>

A considerable part of our research by Dr. Valavani, Professor Athans, and Mr. Rohrs has been focussed on the convergence patterns exhibited by the various adaptive algorithms. Our aim is to obtain a good understanding of the evolution of the adaptive process and, based on this, to improve the rates of convergence, thus making the algorithms feasible for practical applications.

Earlier simulation runs on the digital computer had shown that, although the output errors tend to zero at the end of adaptation, the parameters almost never converge to their "true" values. This is consistent with the "dual effect" which becomes much more pronounced in the general case of <u>stochastic</u> adaptive problems. But even in deterministic adaptive control our knowledge of the precise nature of adaptation is far from satisfactory at the present time.

Systematic simulation studies of a simple second order example at this stage have shown some interesting--rather encouraging--behavior. Bode plots of the controlled adaptive system indicate that it is able to pick up the high frequency (of the model) very fast and track it arbitrarily closely from then on. This points to the existence of an "optimal" choice of auxiliary state variable filters within this framework. Furthermore, the almost identical behavior (in the Bode plots) of the model and controlled plant at the higher--but not cutoff--frequency part of the spectrum may mean that the uncertainty region around the "nominal" plot near the critical point (on the Nyquist plane) is greatly reduced. This has obvious implications for the stability properties of the overall system.

Work is progressing along these lines in order to quantify these regions of uncertainty during adaptation. Optimization of the adaptive process--in terms of improved convergence characteristics--can then be carried out in the sense of minimizing the radii bounding uncertainty around "nominal" trajectories. This will obviously affect the choice of adaptive gains for improved performance.

It has been argued [2] that knowledge of the sign of the high frequency gain--as well as the exact relative degree of the process--provide enough information for the design of a "heuristic" controller with comparable performance but simpler implementation. We are presently trying to "capture" the effect of different assumptions with respect to the high frequency gain on the adaptation. Simultaneously, the pattern of evolution of the characteristic roots (poles) of the controlled system as well as the transfer function configurations to which it evolves will provide valuable information on which to draw for design improvements.

Our simulation studies so far have also shown that the control input exhibits an interesting behavior--particularly when a comparison is made with the evolution pattern of the characteristic roots of the plant. Further, it appears that the adaptive process overall exhibits a rather consistent pattern in terms of frequency characteristics and the "adaptive" pole-zero configuration, which admits to an analytic description. Work is progressing in this direction and the first results will be contained in [3].

5. Stochastic Adaptive Control

When additive and/or multiplicative noise are present in the system, the entire adaptive control problem becomes very complex. To date, there are no complete proofs for its stability. In the model reference and self-tuning regulator approach the disturbance usually enters as observation (additive) noise. However, due to feedback in the adaptive loop it also becomes multiplicative and affects the "estimated" or "controlled" parameter values. So far, stochastic adaptive controllers in this case use some ad hoc modifications of their deterministic counterparts, motivated from the fact that the effects of noise should be ignored (switched-off) in the adjustment of the control parameters after some advanced stage is reached in the adaptation. Stability results in this direction have only been obtained locally and have had to rely on certain restrictive assumptions concerning the boundedness (and smoothness) of functions involved in the proofs.

Our approach to this problem has been a global one. We are trying to formulate a general framework within which the adaptive laws can be chosen so as to guarantee stability of the overall system at the outset. Research has been carried out in an effort by Dr. Valavani to define a "stochastic storage function" concept, motivated from the corresponding one used in hyperstability theory for deterministic systems. The ideas of passivity and positive reality are intimately linked with this. It is interesting to note at this point that Brockett and Willems [4] were able to define a scalar constant quantity which they called "temperature" for stochastic systems whose deterministic parts are positive real. We expect to be able to arrive at a definition of our "energy indexing function" ("stochastic storage functions") from the description of the "uncertainty regions" obtained during adaptation for the stochastic systems. Our results from section 4 are very encouraging in this direction. Given that the deterministic adaptive laws were derived from realizing positive real transfer functions (which are strongly stable) for the error equations, at least in the transient stage, it is not unreasonable to expect that the "uncertainty regions"--which will now depend on noise parameters as well--will be containable and rather well behaved. Within this framework we can then choose our adaptive laws so as to make the "radii" of these "uncertainty regions" decrease as adaptation proceeds. Work is progressing along these lines and will be reported at a future date.

Adaptive Dual Control Studies

Most stochastic optimal control problems are not amenable to a solution through the stochastic dynamic programming equation. This is so because of the "curse of dimensionality." The need, therefore, naturally arises for suboptimal algorithms. Those suboptimal algorithms should, however, share desirable qualitative features with the optimal controls. The study of simple examples of discrete-time linear systems with quadratic cost and multiplicative noise indicates two consequences of parameter uncertainty on the optimal control law. On the one hand, the presence of uncertainty in the system parameters has a stimulating action on the optimal control, because a control exercised at a given time can improve the accuracy of future parameter estimates. This effect has been called loosely the probing effect of the control. On the other hand, the presence of uncertainty which cannot be reduced by the control has an inhibitory, loosely called the caution, effect on the control; the larger those irreducible uncertainties, the more attenuated the control should be. None of these consequences of uncertainties, the so-called dual effect, are captured by the naive "certainty equivalent" (CE) control law, which is obtained by setting all random parameters to their a priori mean values and treating the system as deterministic.

In the more general cases, wide-sense dual adaptive algorithms have been suggested. The crux of those adaptive algorithms is to approximate the cost-to-go in the dynamic programming equation by expanding it about a nominal trajectory to second-order terms in perturbations resulting from random disturbances. The resulting cost, called the dual cost, is minimized to yield the suboptimal control at the corresponding time stage. It has been observed by simulations that the algorithms displayed the desirable caution and probing features. Moreover, it has been claimed that the dual cost could be decomposed in a sum of terms which account respectively for the caution effect, the probing effect and the deterministic part of the cost.

In general, however, it is impossible to compare such dual control laws with the optimal one, which is unknown, in the case of constant but unknown parameters. During the past year Mr. Dersin and Professors Athans and Kendrick [5] considered a different special case of a scalar, discrete-time linear system with white multiplicative gaussian noise and perfectly observed state. The optimal control law of such systems, for a quadratic performance index, is known. We show that, in that special case, it is possible to explicitly derive the dual cost and the dual control in closed form, when the length of the planning horizon goes to infinity.

Some valuable insight can be obtained, since we show that the asymptotic (i.e., infinite horizon) dual control law is in fact equivalent to a first-order expansion of the optimal control law for systems with white parameters as a function of the parameter covariances, about the nominal value of null parameter covariances, which corresponds to a deterministic problem. Since the certainty-equivalent (CE) control is simply a zero-order approximation, the dual control is shown to be intermediate (optimal to linear terms) between the CE and the optimal control. The accuracy of the dual control law for small parameter covariances is quite surprising, as no learning can take place in this problem, due to the white-noise parameter assumption. In other words, if the system parameters have <u>small</u> standard deviations about their mean values, we demonstrate by means of a scalar example that the dual control is (to first order linear terms in the parameter standard deviations) identical to the white-parameter optimal control law, which involves <u>no</u> learning. One can argue both ways whether this is "good news or bad news". The "good news" is that if the system parameters are <u>not</u> very random, then the inherent "robustness" properties of feedback, modulated correctly for parameter uncertainty, require no detailed "learning" of the parameters, provided that certain "caution" is excercised (this is <u>not</u> what the certainty-equivalence principle states). The "bad news" is that the dual control algorithm does not seem to capture the required "caution" effects when the system parameters are <u>very</u> uncertain and very weakly correlated in time.

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1. D. Allinger and S.K. Mitter: New Results on the Innovations Problem for Non-Linear Filtering, LIDS-R-964, January 1980 (submitted to Stochastics).

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This research is concerned with various aspects of stochastic and adaptive systems. Topics covered include: linear and non-linear state estimation, stochastic control and adaptive control.

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